647510 **BSC2MatC201x** **Seat No :________**

B.Sc. Semester - 2 (*CBCS*) Examination

March/April -2019 (New Course)

MATHEMATICS(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

1(A) Answer any one: **(07)**

(1) Get the equation of the tangent plane and normal to the sphere $x^2+y^2+z^2=a^2$ at the point (α,β,γ).

(2) Define: Right Circular Cylinder and find the equation of right circular cylinder with axis $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ and radius r. If the axis of cylinder is X-axis then derive its equation.

(B) Answer any one: **(04)**

(1) Find equation of right circular cylinder whose guiding curve is a circle $x^2+y^2+z^2=9, x-y+z=3$.

(2) Get the equation of tangent plane to the sphere with $A(2,7,6)$ and $B(3,-4,5)$ as extremities of a diameter. Also find the equation of normal to this sphere at the given point A.

(C) Answer any one: **(03)**

(1) Find the area of circle $x^2+y^2+z^2-8x+4y+8z-45=0, x-2y+2z-3=0$.

(2) Find spherical co-ordinates of the point with cylindrical co-ordinates are $\left(2,\frac{7\pi}{6},2\sqrt{3}\right)$.

2 (A) Answer any one: **(07)**

(1) State and prove Euler's theorem for homogeneous function of two variables and deduce that :

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u.$$

(2) (i) For implicit function $f(x,y,z)=0$, Obtain $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(ii) If $f(x,y)=0$ then obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(B) Answer any one: **(04)**

(1) Define: Homogeneous function of two variables and if $u=sin(\sqrt{x}+\sqrt{y})$ then show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}(\sqrt{x}+\sqrt{y})cos(\sqrt{x}+\sqrt{y}).$$

(2) Find f_x and f_y using the definition of partial derivatives at $(0,0)$,

if
$$f(x,y)=\frac{x^2-xy}{x+y};\ (x,y)\neq(0,0)$$
$$=0\quad;\ (x,y)=(0,0)$$

(C) Answer any one: **(03)**

(1) If $f(x,y) = xy\left(\frac{x^2-y^2}{x^2+y^2}\right); (x,y) \neq (0,0)$

$= 0 \quad ; (x,y) = (0,0)$

then discuss the continuity of f at $(0,0)$.

(2) Define: Total differential and if $u = x^4y + y^2z^3$ where $x = rse^t, y = rs^2e^{-t}$ and $z = r^2 sint$ then find the value of $\frac{\partial u}{\partial s}$, when $r = 2, s = 1, t = 0$.

3 (A) Answer any one: **(07)**

(1) State and prove Taylor's theorem.

(2) If $u = \frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$ then the extreme value of u when $lx + my + nz = 0$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ can be obtained by an equation $\frac{l^2a^4}{1-a^2u} + \frac{m^2b^4}{1-b^2u} + \frac{n^2c^4}{1-c^2u} = 0$.

(B) Answer any one: **(04)**

(1) If $f(x,y) = x^2y - 3y$ then find approximate value of $f(5.2, 6.85)$.

(2) Define: Jacobian and if $u = x^2 + y^2 + z^2, v = xy + yz + zx$, $w = x^3 + y^3 + z^3 - 3xyz$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$.

(C) Answer any one: **(03)**

(1) Define: Local maxima and minima and find local maxima and minima for the function (if possible), $f(x,y) = x^2 - 2xy + 2y^2 - 2x + 2y + 1$

(2) If there is 0.05% error obtain in measurement of length of sides of rectangle then what should be error in the measurement of its area.

4 (A) Answer any one: **(07)**

(1) Define: Adjoint of a matrix and if $A = [a_{ij}]_n, B = [b_{ij}]_n$ then prove that:

$(i) A \cdot (adjA) = (adjA) \cdot A = |A|I$

$(ii) (adjA)^T = adjA^T$

(2) Define: Row and Column rank of a matrix, prove that row rank and column rank of a matrix is same as its rank.

(B) Answer any one: **(04)**

(1) Find two non-singular matrices P and Q such that PAQ is in the normal form, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$, Also find the rank of the matrix A.

(2) Show that every square matrix is uniquely expressible as the sum of Hermitian and skew Hermitian matrix.

(C) Answer any one: **(03)**

(1) Define: (i) Involutory matrix (ii) Orthogonal matrix

Also prove that: $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$ is involutory matrix

and $B = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$ is orthogonal matrix.

(2) Find inverse of the matrix $A = \begin{bmatrix} 5 & 8 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ using preliminary row operations.

5 (A) Answer any one: **(07)**

(1) State and prove Cayley-Hamilton theorem.

(2) Define: Eigen value and Eigen vector of matrix and prove that

(i) There is unique eigen value with respect to any eigen vector of a given matrix.

(ii) There is more than one eigen vectors with respect to any eigen value of a given matrix.

(B) Answer any one: **(04)**

(1) Solve the system of equations:

$x + 3y + 2z = 0, x + 4y + 3z = 0, x + 5y + 4z = 0$

(2) Find Eigen value and eigen vectors of the matrix

$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(C) Answer any one: **(03)**

(1) Solve: $5x + 2y + 2z = 17, 9x + 4y + 2z = 27.$

(2) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial.

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